Sistemi determinati, indet., imp.

$$\begin{cases} ax+by=c \\ a'x+b'y=c' \end{cases}$$

$$\begin{cases} x+y=3 \\ x+y=2 \end{cases} \qquad S=\emptyset$$

$$\begin{cases} x+y=1 \\ 2x+2y=2 \end{cases} \qquad \begin{cases} \cancel{x+y=1} \\ x+y=1 \end{cases} \qquad S=\mathbb{R}$$

1) $\dfrac{a}{a'}=\dfrac{b}{b'}=\dfrac{c}{c'}$ $\qquad S=\mathbb{R}$

2) $\dfrac{a}{a'}=\dfrac{b}{b'}\neq\dfrac{c}{c'}$ $\qquad S=\emptyset$

3) $\dfrac{a}{a'}\neq\dfrac{b}{b'}$ $\qquad$ determinato

$$D = \begin{vmatrix} a & b \\ a' & b' \end{vmatrix} = ab' - a'b$$

$$D_x = \begin{vmatrix} c & b \\ c' & b' \end{vmatrix} = cb' - c'b$$

$$D_y = \begin{vmatrix} a & c \\ a' & c' \end{vmatrix} = ac' - a'c$$

$$x = \frac{D_x}{D} = \frac{cb' - c'b}{ab' - a'b}$$

$$y = \frac{D_y}{D} = \frac{ac' - a'c}{ab' - a'b}$$

$$D = 0 \iff ab' - a'b = 0$$

$$ab' = a'b \iff \frac{a}{a'} = \frac{b}{b'}$$

$$D_x = 0 \iff cb' - c'b = 0$$

$$cb' = c'b \iff \frac{c}{c'} = \frac{b}{b'}$$

Metodo di riduzione

$$\begin{cases} 2x + y = 5 \\ 3x - y = 10 \end{cases}$$

$$5x \quad // = 15 \implies x = 3$$

$$\begin{array}{r|l} 3 & 2x + y = 5 \\ -2 & 3x - y = 10 \end{array}$$

$$\begin{cases} 6x + 3y = 15 \\ -6x + 2y = -20 \end{cases}$$

$$// \quad 5y = -5$$

$$y = -1$$

$$\begin{cases} 6x + 3y = 15 \\ 6x - 2y = 20 \end{cases}$$

$$// \quad 5y = -5$$

Sistemi letterali

$$\begin{cases} ax + 3ay = 1 \\ x + 2y = -2 \end{cases}$$

$$D = \begin{vmatrix} a & 3a \\ 1 & 2 \end{vmatrix} = 2a - 3a = -a$$

$$D_x = \begin{vmatrix} 1 & 3a \\ -2 & 2 \end{vmatrix} = 2 + 6a$$

$$D_y = \begin{vmatrix} a & 1 \\ 1 & -2 \end{vmatrix} = -2a - 1$$

Se $D \neq 0$, cioè se $-a \neq 0$, ovvero se $a \neq 0$, il sistema è determ. e la soluzione è data da:

$$x = \frac{D_x}{D} = \frac{2+6a}{-a} = -\frac{2+6a}{a}$$

$$y = \frac{D_y}{D} = \frac{-2a-1}{-a} = \frac{2a+1}{a}$$

Se $D = 0$, cioè se $a = 0$, si ha:

$D_x = 2$ $\qquad$ $D_y = -1$

$S = \emptyset$

$$\begin{cases}(a+1)x-ay=a\\ax+ay=0\end{cases}$$

$$D=\begin{vmatrix}a+1 & -a\\ a & a\end{vmatrix}=a^2+a+a^2=$$
$$=2a^2+a=\underline{a(2a+1)}$$

$$D_x=\begin{vmatrix}a & -a\\ 0 & a\end{vmatrix}=\underline{a^2}$$

$$D_y=\begin{vmatrix}a+1 & a\\ a & 0\end{vmatrix}=\underline{-a^2}$$

Se $D\neq 0$, cioè se $a(2a+1)\neq 0$, ovvero se $\underline{a\neq 0} \wedge \underline{a\neq -1/2}$, si ha:

$$x=\frac{a^{\cancel{2}}}{\cancel{a}(2a+1)}=\frac{a}{2a+1} \qquad y=\frac{-a^{\cancel{2}}}{\cancel{a}(2a+1)}=-\frac{a}{2a+1}$$

Se $a = 0$ si ha:

$D = 0$, $D_x = 0$, $D_y = 0$

$S = \mathbb{R}$

Se $a = -1/2$ si ha:

$D = 0$, $D_x = 1/4$, $D_y = -1/4$

$S = \emptyset$